

Philosophy 211 -- Assignment #3

I. Prove these sequents.

1. $((S \rightarrow T) \rightarrow R) \rightarrow Q, P \rightarrow S \vdash ((P \rightarrow T) \rightarrow R) \rightarrow Q$
2. $(T \rightarrow Q) \rightarrow (P \rightarrow R), \sim Q \rightarrow \sim S, \sim P \rightarrow \sim M \vdash M \rightarrow ((T \rightarrow S) \rightarrow R)$
3. $P \rightarrow (\sim R \rightarrow Q) \vdash (P \& \sim Q) \rightarrow R$
4. $P \rightarrow \sim Q, \sim R \rightarrow S \vdash (P \& \sim S) \rightarrow (\sim Q \& R)$
5. $((P \& Q) \& R) \rightarrow S \vdash R \rightarrow (P \rightarrow (\sim S \rightarrow \sim Q))$
6. $\sim P \vee Q \vdash (R \vee \sim Q) \rightarrow (P \rightarrow R)$
7. $(S \rightarrow T) \rightarrow U \vdash ((S \vee Q) \rightarrow T) \rightarrow U$
8. $\sim(P \vee Q) \vee ((P \vee T) \rightarrow S) \vdash P \rightarrow ((Q \rightarrow U) \vee S)$
9. $((Q \vee R) \& \sim S) \rightarrow T, Q \& U \vdash (\sim S \vee \sim U) \rightarrow (T \& U)$
10. $(Q \vee T) \vee U, \sim U \rightarrow \sim S, \sim U \vee Q \vdash (\sim Q \rightarrow T) \& (S \rightarrow Q)$

II. Complete these proofs. I have identified all of the assumptions in each problem.

1. $((A \rightarrow B) \rightarrow C) \rightarrow D, E \rightarrow A,$
 $F \rightarrow C \vdash ((\sim B \rightarrow \sim E) \rightarrow F) \rightarrow D$

2. $E \vee (T \& L), M \rightarrow \sim(T \vee H),$
 $U \& \sim E \vdash (L \& \sim M) \& U$

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|---|--|---|
| 1 | (1) $((A \rightarrow B) \rightarrow C) \rightarrow D$ | A |
| 2 | (2) $E \rightarrow A$ | A |
| 3 | (3) $F \rightarrow C$ | A |
| 4 | (4) $(\sim B \rightarrow \sim E) \rightarrow F$ | A |
| 5 | (5) $A \rightarrow B$ | A |
| 6 | (6) $\sim B$ | A |
| | (7) $\sim A$ | |
| | (8) $\sim E$ | |
| | (9) $\sim B \rightarrow \sim E$ | |
| | (10) F | |
| | (11) C | |
| | (12) $(A \rightarrow B) \rightarrow C$ | |
| | (13) D | |
| | (14) $((\sim B \rightarrow \sim E) \rightarrow F) \rightarrow D$ | |

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|---|------------------------------------|---|
| 1 | (1) $E \vee (T \& L)$ | A |
| 2 | (2) $M \rightarrow \sim(T \vee H)$ | A |
| 3 | (3) $U \& \sim E$ | A |
| | (4) $\sim E$ | |
| | (5) $T \& L$ | |
| | (6) L | |
| | (7) T | |
| | (8) $T \vee H$ | |
| | (9) $\sim M$ | |
| | (10) U | |
| | (11) $L \& \sim M$ | |
| | (12) $(L \& \sim M) \& U$ | |

TURN OVER!!!

III. Which of the following are satisfactory representations in sentential logic of the following English sentence?

At most one of Bill, Mary, and Tom will attend the meeting.

- A. $\sim(B \vee M) \& (\sim(B \vee T) \& \sim(M \vee T))$
- B. $(\sim B \& \sim M) \vee ((\sim B \& \sim T) \vee (\sim T \& \sim M))$
- C. $(\sim B \rightarrow (\sim M \vee \sim T)) \& ((\sim M \rightarrow (\sim B \vee \sim T)) \& (\sim T \rightarrow (\sim B \vee \sim M)))$
- D. $(B \rightarrow \sim(M \vee T)) \& (M \rightarrow \sim(B \vee T))$
- E. $\sim((B \& M) \vee ((B \& T) \vee (M \& T)))$

IV. For this question, I am operating under the same assumptions as in the Knight-Knave problem from the first assignment. Suppose that in the country of Knights and Knaves you meet three individuals, A, B, and C. You discover that at least one of them is a Knight and at least one of them is a Knave. Then A says “B or C is a Knight” and B says “A or C is a Knight.”

Which of them are Knights and which are Knaves? Explain your reasoning.